

Electron-electron interactions in a weakly screened two-dimensional electron system

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We probe the strength of electron-electron interactions using magnetoconductivity measurements of two-dimensional non-degenerate electrons on liquid helium at 1.22 K. Our data extend to electron densities that are two orders of magnitude smaller than previously reported. We span both the independent-electron regime where the data are qualitatively described by the self-consistent Born approximation (SCBA), and the strongly-interacting electron regime. At finite fields we observe a crossover from SCBA to Drude theory as a function of electron density.

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Electrons supported by a liquid-helium surface form a low density, non-degenerate two-dimensional (2D) electron gas. Aside from the non-degeneracy, it differs from other 2D electron systems in the strength of the electron-electron (e-e) interaction¹⁻⁵. The Coulomb interaction is weakly screened by metallic plates that are separated from the electron layer by about 1 mm. It is an ideal system for testing the properties of strongly-interacting electrons.

One of the interesting properties of this non-degenerate 2D electron gas is the density of states (DOS) in a magnetic field. The DOS peaks at the Landau levels (LLs) have a width that depends both on the scattering rate and the e-e interaction. The width of the DOS peaks has been calculated^{6,7} in the self-consistent Born approximation (SCBA) and has been studied experimentally^{2-5,8-11} through measurements of the magnetoconductivity. We report magnetoconductivity measurements from an extremely low density, $\sim 1.9 \times 10^9 \text{ m}^{-2}$, where e-e interactions are negligible, to densities where Coulomb interactions dominate the width of the DOS peaks.

In this system, electron-helium atom scattering dominates at temperatures above 0.8 K, while electron-ripplon scattering is important at lower temperatures. Each collision between an electron and a helium atom changes the electron energy by about one percent. Thus, scattering is quasi-elastic. The helium atoms in the vapor act as short range-scatterers to a very good approximation.

In zero magnetic field the density of states is constant: $D_0(E) = m/\pi\hbar^2$. The magnetoconductivity of electrons, $\sigma_{xx}(B)$, is given in the Drude model for $\mu_0 B \ll 1$ as

$$\sigma_{xx}(B) = \frac{\sigma_{xx}(0)}{[1 + (\mu_0 B)^2]}, \quad \sigma_{xx}(0) = ne\mu_0, \quad (1)$$

where $\sigma_{xx}(0)$, $\mu_0 = e\tau_0/m$, and τ_0 are the zero field conductivity, mobility and scattering time, respectively. The Drude model assumes that electrons are independent and move in straight paths (in zero field) and in classical orbits (in a magnetic field).

A magnetic field transverse to the 2D electrons changes the density of states, and consequently the magnetoconductivity of electrons dramatically. In a magnetic field, Landau levels separate when $\Delta/\hbar\omega_c \sim 1$. Here, Δ is the

width of the Landau level. The broadening Δ_a due to collisions with helium atoms and the broadening Δ_e due to electron-electron interactions enter the Δ as⁵

$$\Delta^2 = \Delta_a^2 + \Delta_e^2. \quad (2)$$

Once LLs form, the Drude model loses its validity and a crossover to the SCBA regime occurs. In SCBA, the DOS and thus the magnetoconductivity is obtained self consistently in the Born approximation. The broadening Δ_a has been calculated^{6,7} in the SCBA limit for a semi-elliptic DOS and short range scatterers and given by

$$\Delta_a = \frac{\hbar}{\tau_B} = \hbar \left(\frac{8}{\pi} \frac{\omega_c}{\tau_0} \right)^{1/2}, \quad (3)$$

where $\omega_c = eB/m$ is the cyclotron frequency and τ_B^{-1} is the scattering rate in a magnetic field. For $\Delta_e \rightarrow 0$ and $\hbar\omega_c < \Delta_a$, we assume that the broadening Δ_a is determined by the zero field scattering time and is on the order of $\sim \hbar/\tau_0$. The crossover is delayed by many electron effects^{2,4,5} as seen in Eq. 2. The broadening Δ_e is given by theory⁵ as

$$\Delta_e = eE_f\lambda_T; \quad E_f \approx \left(\frac{11kTn^{3/2}}{4\pi\epsilon\epsilon_0} \right)^{1/2}, \quad (4)$$

where $\bar{\epsilon} = (\epsilon_{He} + 1)/2 = 1.028$, E_f is the fluctuating field^{1,2} an electron feels due to redistribution of other electrons as it moves, and the thermal wavelength λ_T is the characteristic size of an electron in the classical limit $\hbar\omega_c < kT$, which holds for our experimental data.

The SCBA is valid for static scatterers and a vanishing coherence time^{7,12}. For electrons on helium the coherence time is on the order of τ_0 , and the scattering is dynamic, i.e., the vapor atoms move slowly. For this case the behavior should be qualitatively given by the SCBA expression.

The crossover to the SCBA regime has already been confirmed by earlier experiments in the quantum limit where $\hbar\omega_c > kT$, whereas no such crossover has been reported in the classical limit. This resulted from the electron densities being too high (greater than 10^{11} m^{-2}) in

previous experiments.^{2,4,8-11} At those densities the LLs can separate only at very high magnetic fields due to many electron effects. In our experiment, we were able to reduce the electron density another two orders of magnitude to $1.9 \times 10^9 \text{ m}^{-2}$. At this density, the plasma parameter $\Gamma = e^2(\pi n)^{1/2}/4\pi\epsilon_0 kT$, which is the ratio of the unscreened Coulomb energy to the thermal kinetic energy, is ~ 1.1 compared to ~ 13 at $n \sim 10^{11} \text{ m}^{-2}$. Thus we were able to observe, for the first time, the SCBA magnetoconductivity in classical fields $\hbar\omega_c < kT$.

In Fig. 1. we give a qualitative picture of broadening of a semi-elliptic LL by electron-electron interactions. The quantity Δ_e is, in fact, the uncertainty in the energy of an electron due to its finite size λ_T in a field E_f and depends strongly on the electron density via E_f . The theory predicts that the broadening Δ_e is on the order of Δ_a for $\mu_0 = 25 \text{ m}^2/\text{Vs}$ and $n \sim 10^{11} \text{ m}^{-2}$.

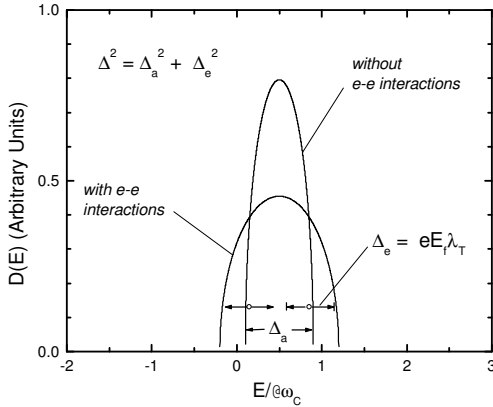


FIG. 1. A qualitative picture of broadening of a LL due to many electron effects. The additional uncertainty Δ_e , depicted by double arrows in the figure, broadens the width. The total broadening is given by Δ .

An expression for the magnetoconductivity of 2D non-degenerate electrons has been calculated by van der Heijden et. al.⁸ in SCBA regime and given for a semi-elliptic density of states by

$$\sigma_{xx}(B) = \frac{2 \coth(\hbar\omega_c/kT)}{\pi I_1(\Delta_a/2kT)} f(\Delta_a, T);$$

$$f(\Delta_a, T) = [\cosh(\frac{\Delta_a}{2kT}) - \frac{\Delta_a}{2kT} \sinh(\frac{\Delta_a}{2kT})] \frac{ne}{B}, \quad (5)$$

where I_1 is the modified Bessel function of order 1, n is the electron density, and Δ_a is the width of Landau levels. Eq. 5 is valid when $\Delta_e \ll \Delta_a$, i.e. when $\Delta \rightarrow \Delta_a$.

For $\hbar\omega_c, \Delta_a \ll kT$, Eq. 5 gives

$$\frac{\sigma_{xx}(0)}{\sigma_{xx}(B)} = \frac{3\pi^{3/2}}{8\sqrt{2}} (\mu_0 B)^{3/2}. \quad (6)$$

Here, we emphasize the $B^{3/2}$ dependence of the SCBA magnetoconductivity compared to the B^2 dependence of the Drude magnetoconductivity.

We measured the inverse magnetoconductivity $1/\sigma_{xx}(B)$ of electrons as a function of a magnetic field perpendicular to the 2D electron layer in a Corbino geometry. Electrons were deposited over a $\sim 1 \text{ mm}$ thick helium film from a gaseous discharge. The density of electrons was controlled by carefully adjusting the dc holding voltage on the Corbino electrodes below the liquid helium surface.

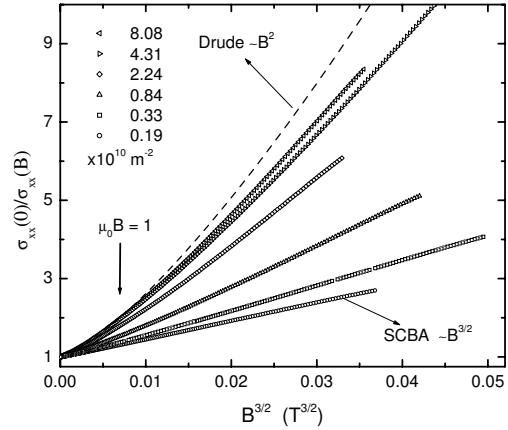


FIG. 2. Normalized inverse-magnetoconductivity plotted as a function of $B^{3/2}$ at $T = 1.22 \text{ K}$. The densities are in units of 10^{10} m^{-2} , and $\mu_0 = 27.5 \text{ m}^2/\text{Vs}$.

In Fig. 2., we show the normalized inverse magnetoconductivity $\sigma_{xx}(0)/\sigma_{xx}(B)$ as a function of $B^{3/2}$ for six electron densities. We see a crossover from the SCBA magnetoconductivity ($B^{3/2}$ dependence) to the Drude magnetoconductivity (B^2 dependence) as the electron density is increased for $\hbar\omega_c/\Delta > 1$. The dashed line is the normalized theoretical Drude magnetoconductivity calculated using the experimental parameters of the highest density curve. We obtain the zero-field mobility and the density for the highest density curve from a fit to the Drude theory in small fields. Then, we calculate the electron densities for the lower density curves using the measured zero-field resistivity $1/ne\mu_0$ and assuming the same zero-field mobility of $\mu_0 = 27.5 \text{ m}^2/\text{Vs}$ obtained for the highest density curve. The fact, that the magnetic field region $\hbar\omega_c/\Delta < 1$ within which the Drude-like behavior is observed gets smaller as the electron density is reduced, prevented us from obtaining the zero-field mobilities accurately and the densities independently for the lower density curves.

The effect of LL broadening due to e-e interactions is seen clearly in the figure as the crossover to the SCBA regime occurs at a lower magnetic field as the electron density is reduced. The $B^{3/2}$ dependence is clear in the curve with the lowest electron density of $0.19 \times 10^{10} \text{ m}^{-2}$.

Note that, in our system, the quantum limit is reached when $B = 0.91$ Tesla. The values of $\hbar\omega_c/kT$ for all the data shown were less than 0.12, and the magnetic fields used fall well into the classical regime.

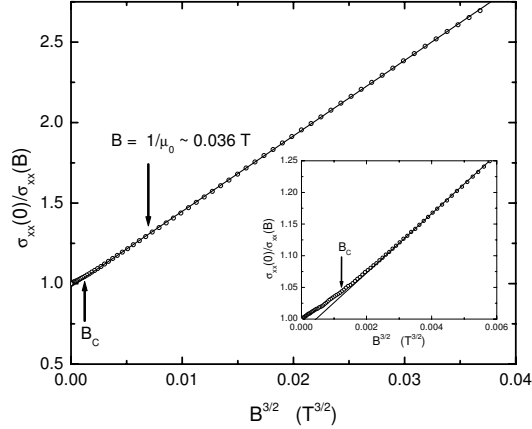


FIG. 3. Normalized inverse-magnetoconductivity vs. $B^{3/2}$ for the lowest density $n = 0.19 \times 10^{10} \text{ m}^{-2}$. $T = 1.22 \text{ K}$.

We plot the curve with the lowest density in Fig. 3. Except for very low fields the data follow SCBA. The field B at which $\mu_0 B = 1$ and the crossover field B_c are shown with arrows. We explain how we obtain the field $B_c \sim 0.011 \text{ T}$ later in the text. The crossover is seen more clearly in the inset of the figure where we show a blow up of the same graph at small fields. The field B_c corresponds to $\sim 30\%$ of the theoretical value $1/\mu_0 \sim 0.036 \text{ T}$ given for no e-e broadening. The solid line is the theoretical fit to Eq. 6. The fit gives a zero field mobility of $\sim 10 \text{ m}^2/\text{Vs}$ which is smaller than the value $27.5 \text{ m}^2/\text{Vs}$. This is in agreement with a lower value of the mobility obtained from fits to the SCBA formula in earlier measurements reported in the quantum limit.⁸

Our data would appear to disagree with theory, which predicts that the contribution Δ_e to the total broadening Δ becomes negligible at electron densities below 10^{11} m^{-2} and that the crossover to SCBA should occur at fields $\sim 1/\mu_0$. The SCBA magnetoconductivity is not fully developed at $B = 0.1 \text{ T}$ even at the low density of $0.84 \times 10^{10} \text{ m}^{-2}$ as a $\ln(\sigma_0/\sigma_{xx})$ vs. $\ln(\mu_0 B)$ graph yields a slope of $\gtrsim 1.7$ which is still greater than the value of 1.5 in SCBA. The contribution Δ_e to the total broadening is more significant than predicted by theory.

In order to obtain a quantitative result for the crossover field, we plot our data as $[\sigma_{xx}(0)/\sigma_{xx}(B)] - 1$ and fit it with the following function $F(\mu_0, B, B_c)$

$$F(\mu_0, B, B_c) = [1 - C(B, B_c)](\mu_0 B)^2 + \frac{3\pi^{3/2}}{8\sqrt{2}} C(B, B_c)(\mu_0 B)^{3/2}, \quad (7)$$

where μ_0 and B_c are the free parameters. The $C(B, B_c)$

is a rapidly-changing crossover function and we find an excellent fit to the low-density data by choosing

$$C(B, B_c) = \tanh^{1/2}\left(\frac{B}{4B_c}\right). \quad (8)$$

This function is 0.5 at $B = B_c$. The fitting function F starts in the Drude regime at $B = 0$ and goes into SCBA at a magnetic field characterized by B_c :

$$F(\mu_0, B, B_c) = (\mu_0 B)^2, \quad B \ll B_c \\ = \frac{3\pi^{3/2}}{8\sqrt{2}} (\mu_0 B)^{3/2}, \quad B \gg B_c. \quad (9)$$

The values of B_c obtained from the fits are a measure of the actual crossover field and thus the width Δ , but do not necessarily represent the actual widths of the LLs, since the overlap between neighboring levels at the actual crossover field is not known exactly and the fitting function has no theoretical basis.

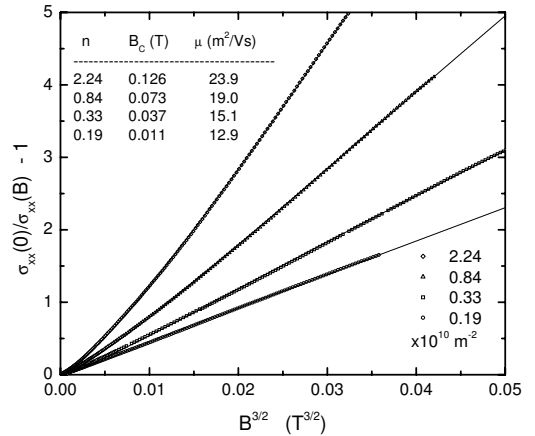


FIG. 4. The four lowest-density-curves with fits to the function $F(\mu_0, B, B_c)$ vs. $B^{3/2}$.

In Fig. 4., we show the curves with their non-linear least square fits, shown by the solid lines, to Eq. 7. The best fits are obtained allowing μ_0 and B_c to be free parameters. We leave the curves with the highest two densities out of this analysis since, for those data, the crossover region extends too far beyond the magnetic fields used in the experiments, and this results in an error in the fitting parameter B_c .

The values of B_c obtained from the fits above give an approximate width Δ^* of the LLs for each electron density. We set $\Delta^* = \hbar\omega_c(B = B_c) = \hbar e B_c / m$, and plot the values of Δ^* as a function of electron density in Fig. 5. In order to compare with the theoretical expression for the width of the LLs, we plot Eq. 2 as a solid line in the figure for $\Delta_a = 15 \text{ mK}$ and $\Delta_e = 11eE_f\lambda_T$. Although the values of Δ^* obtained from the fits differ from the theoretical values by a factor of 11, they give the correct functional dependence on the electron density.

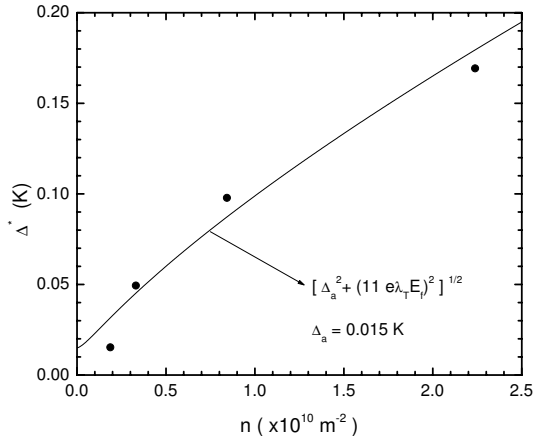


FIG. 5. The values of Δ^* as a function of electron density. The solid line is described in the text.

The values of μ_0 from the fits correspond to a reduced mobility for low densities. In our initial fits to the data we allowed for different mobilities in the two regimes as adjustable parameters. The best fits were given with equal mobilities for the two regimes. At higher densities van der Heijden et al.⁸ deduced a mobility from the SCBA regime that was a factor of two smaller than the Drude mobility. However, our fits required an adjustment of the Drude mobility as well. The smaller apparent mobility occurs in part, because the low-density data is weighted by the SCBA regime, and we observe no well-developed Drude regime at low densities. Deviations from Drude behavior begin at zero field in our data. Despite this fact, the strong variation in the field where the data crossover to the SCBA regime is apparent in both the data curve shown in Fig. 2 and the fits shown in Fig. 4. The crossover function $C(B, B_c)$ under-weighs the Drude part, but fits very well because there is no well developed Drude regime at very low densities in our data.

In conclusion, we measured the magnetoconductivity of non-degenerate electrons in the very low-density limit. The effect of e-e interactions is clearly demonstrated in these data. Electron-electron interactions have a significant effect on the magnetoconductivity causing a delay in the transition from Drude to SCBA regime as a function of a magnetic field. When the many electron effects are

negligible the transition is observed in classical fields. We studied the dependence of the LL width on the electron density for the first time. Our results agree with the theory qualitatively, but differ by a large numerical factor. A more detailed theoretical analysis of classical electron scattering from dynamic scatterers in a magnetic field is required to reconcile experiment and theory.

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